Fractional quantum Hall states of atoms in optical Lattices

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We describe a method to create fractional quantum Hall states of atoms confined in optical lattices. We show that the dynamics of the atoms in the lattice is analogous to the motion of a charged particle in a magnetic field if an oscillating quadrupole potential is applied together with a periodic modulation of the tunneling between lattice sites. We demonstrate that in a suitable parameter regime the ground state in the lattice is of the fractional quantum Hall type and we show how these states can be reached by melting a Mott insulator state in a super lattice potential. Finally we discuss techniques to observe these strongly correlated states.

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Ultra-cold atomic gasses [1] provide a unique access to quantum many body systems with well understood and controllable interactions. Whereas most of the experiments in this field have been carried out in the regime of weak interactions, the recent achievements involving Feshbach resonances [2] and the realization of a Mott-insulator state of atoms in optical lattices [3, 4] enters into the regime of strong interaction with a richer and more complex many body dynamics. At the same time a realization of strongly correlated states of fractional quantum Hall type [5] has recently been suggested in cold atomic gases [6]. These proposals involve atoms in rotating harmonic traps, which mimic the effective magnetic field. However, weak interaction between the particles (and correspondingly small gap in the excitation spectrum), required precision on trap rotation and finite temperature effects make these proposals difficult to realize experimentally. In this Letter we present a novel method that uses atoms in optical lattices to create states of the fractional quantum Hall type. Since the interactions of atoms localized in the lattices are strongly enhanced compared to the interaction of atoms in free space, these states are characterized by large energy gaps.

The fractional quantum Hall effect occurs for electrons confined to a two dimensional plane (the xy plane) in a strong magnetic field. In the simplest form the effect occurs if the number of magnetic fluxes N_ϕ (measured in units of the fundamental flux quanta $\Phi_0 = 2\pi\hbar/e$) is an integer m times the number of particles $N_\phi = m \cdot N$. At this value of the magnetic field the ground state of the system is an incompressible quantum liquid which is separated from all other states by an energy gap and is well described by the Laughlin wavefunction [7]

$$\Psi(z_1, z_2, ...z_N) = \mathrm{e}^{\left(-\sum_j |z_j|^2/4\right)} \prod_{j<k} (z_j - z_k)^m, \qquad (1)$$

where $z = x + iy$, and where we have assumed the symmetric gauge and suitable magnetic units. Due to the Pauli principle only the states with odd (even) m is applicable to fermions (bosons). In this Letter we for simplicity only consider bosons and $m = 2$.

Below we describe a method to produce the states (1) for a system of ultra-cold atoms in an optical lattice. In such a system these states are protected by an excitation gap which is controlled by the tunneling energy. For typical experimental parameters this is much larger than the energy scale in the macroscopic magnetic traps considered previously [6]. The larger energy gap is a clear advantage from an experimental point of view because the state is more robust to external perturbations, and the present approach could therefore enable the realization of the Laughlin state with a larger number of particles. We note further that experiments with periodically modulated quantum Hall probes [8] and tunnel coupled superconducting islands [9] are trying to reach the regime studied here, and the extension of the quantum Hall physics to a lattice system is therefore an interesting non-trivial problem in its own right. For non-interacting particles the energy spectrum on the lattice (the so-called Hofstadter butterfly [10]) is very different from that of Landau levels. We nevertheless show that at least in a certain parameter regime, the Laughlin state provides a reasonable description for the resulting many-body physics of the strongly interacting system.

We consider atoms trapped in a square two-dimensional optical lattice. If the atoms in the lattice are restricted to the lowest Bloch-band the system can be described by a Bose-Hubbard Hamiltonian [11]

$$H = -J \sum_{\{j,k\}} (\hat{a}_j^\dagger \hat{a}_k + \hat{a}_k^\dagger \hat{a}_j) + U \sum_j n_j(n_j - 1), \qquad (2)$$

where the first sum is over neighboring sites j and k, $\hat{a}_j$ and $n_j = \hat{a}_j^\dagger \hat{a}_j$ are the boson annihilation and number operators on site j, J is the tunneling amplitude, and U is the onsite interaction energy. The individual lattice sites will below be specified by a pair of integers x and y.

An essential ingredient in our proposal is an effective magnetic field for neutral atoms in optical lattices. Different approaches which attain this goal has already been proposed [12], but here we present an alternative procedure that may simplify the experimental realization.

Our procedure involves a combination of a time-varying quadrupolar potential $V(t) = V_{\rm qp}\sin(\omega t)\cdot\hat{x}\cdot\hat{y}$, and a modulation of the tunneling in time. The tunneling between neighboring sites decreases exponentially with the intensity of the lasers creating the lattice, whereas the shape of the wavepacket (the Wannier functions) has a much weaker dependence [11]. By varying the laser intensity the tunneling can therefore be varied rapidly in time. Assume that the tunneling in the x-direction is turned on for a short period around $\omega t = \pi\cdot 2n$, $(n = 0, 1, 2...)$ and that the tunneling in the y-direction is turned on for a short period around $\omega t = \pi\cdot(2n+1)$ (see Fig. 1 (a)). As illustrated by the simplified picture in Fig. 1 (b) and (c) such a time sequence creates an effective Lorentz force (magnetic field) in the lattice. This can be shown mathematically by assuming that the tunneling is only present in a very short time interval τ. The total evolution after m periods is then given by (neglecting for now the interaction between the particles)

$$U\left(t=\frac{m2\pi}{\omega}\right) = \Big(\mathrm{e}^{-i\tau T_x/2\hbar}\mathrm{e}^{2iV_{\rm qp}\hat{x}\hat{y}/\omega\hbar}\mathrm{e}^{-i\tau T_y/\hbar}\times \mathrm{e}^{-2iV_{\rm qp}\hat{x}\hat{y}/\omega\hbar}\mathrm{e}^{-i\tau T_x/2\hbar}\Big)^m, \tag{3}$$

where T_x and T_y are the kinetic energy operators describing tunneling in the x and y direction respectively. By using $\exp(i2\pi\alpha\hat{x}\hat{y})\hat{a}^\dagger_{x,y+1}\hat{a}_{x,y}\exp(-i2\pi\alpha\hat{x}\hat{y}) = \hat{a}^\dagger_{x,y+1}\hat{a}_{x,y}\exp(i2\pi\alpha x)$ with $\alpha = V_{\rm qp}/\pi\hbar\omega$ this expression can be reduced to the time evolution $U = \exp(-iH_{\rm eff}t/\hbar)$ from an effective Hamiltonian

$$H_{\rm eff} \approx -J\sum_{x,y}\hat{a}^\dagger_{x+1,y}\hat{a}_{x,y} + \hat{a}^\dagger_{x,y+1}\hat{a}_{x,y}\mathrm{e}^{i2\pi\alpha x} + \mathrm{H.C.}, \tag{4}$$

where the tunneling strength J is the average tunneling per period, and where we have omitted terms of order $J(J/\omega)^2$. Eq. (4) describes the behavior of a charged particle on a lattice with a magnetic flux $\alpha\Phi_0$ going through each unit cell, and hence the procedure introduces an effective magnetic field in the lattice. The gauge in Eq. (4) (Landau gauge) is determined by the time we terminate the sequence in Fig. 1 (a) and a different gauge would appear if we terminated at a different time.

We now turn to the fractional quantum Hall effect for strongly interacting atoms in the presence of an effective magnetic field. In the limit of small α and a small number of atoms per lattice site the dynamics of the system reduces to the continuum limit of particles in a magnetic field with infinitely short range interactions. If the interaction is repulsive, the Laughlin wavefunction (1) is known to be the absolute ground state of the system when $N_\phi = m\cdot N$ [13]. This limit corresponds to a very low density gas and therefore the energy gap between the ground and excited states is vanishingly small (see below). To extend this analysis to the situation when α is no longer vanishingly small we have performed a direct numerical diagonalization of the Hamiltonian in Eq. (4) for a small number of hard-core bosons (corresponding to $J \ll U$). (In different contexts, similar problems were considered in Ref. [14]).

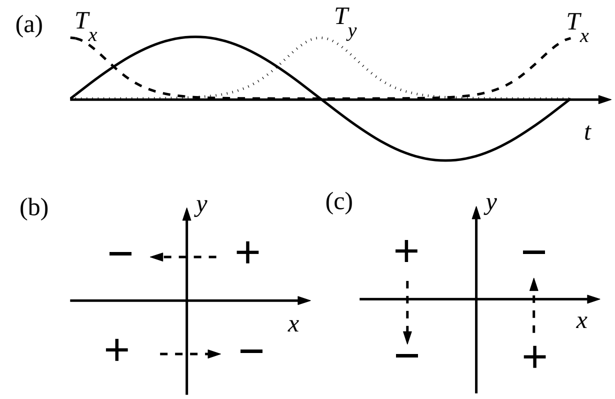

FIG. 1: (a) One period of the sequence used to create an effective magnetic field. The time evolution of the quadrupole potential is shown by the full curve and the dashed and dotted lines indicate the tunneling in the x and y direction. For illustration the shown evolution of the tunneling is obtained from a sinusoidal variation of the lattice potential between 5 and 40 recoil energies [11]. (b) and (c) Physical explanation of the procedure. (b) Tunneling in the x-direction is followed by a positive potential in the first and third quadrant (signs in the figure), and hence atoms will experience a lower potential by moving in the direction of the dashed arrows. (c) Same as (b) but with tunneling in the y-direction and opposite sign of the potential. When combined the dashed lines in (b) and (c) makes a circular cyclotron motion.

To investigate the effect of finite α we fix the number of fluxes and particles so that $N_\phi = 2N$ and vary α by changing the size of the lattice. (We only consider lattices where the size in the x and y direction are the same or differ by unity). In Fig. 2 (a) we show the overlap of the ground state wavefunction from the diagonalization of the Hamiltonian with the Laughlin wavefunction (1). For $\alpha \lesssim 0.3$ the ground state has a very good overlap with the Laughlin wavefunction. We have assumed periodic boundary conditions in order to represent the bulk properties of a large optical lattice. For the situation considered here ($m = 2$) the combination of periodic boundary conditions and a magnetic field gives rise to a two fold degeneracy of the ground state in the continuum limit ($\alpha \ll 1$), where the two ground states only differ by their center of mass wavefunctions [15]. The symmetry analysis leading to this degeneracy does not apply in the presence of the lattice (for a discussion of the symmetries in a periodic potential see Ref. [16]). For all points in Fig. 2 (except the point $N = 5$, $\alpha = 1/3$), however, the diagonalization gives two almost degenerate ground states which are separated from the excited states (see Fig. 3 (a) at $V_0 = 0$). The periodic generalization of the Laughlin wavefunction [17] also have a two-fold center of

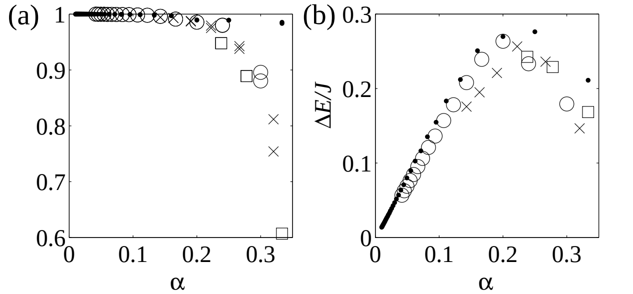

FIG. 2: (a) Overlap of the ground state wavefunction with the Laughlin wavefunction (1). (b) Energy gap ΔE to the lowest excited state. In the figure we have fixed the number of particles and fluxes ($N_\phi = 2N$) and vary the flux per unit cell α by varying the size of the lattice. The shown results are for $N = 2$ (•), $N = 3$ (∘), $N = 4$ (×), and $N = 5$ (□).

mass degeneracy and in Fig. 2 (a) we show the overlap of the two lowest states from the diagonalization with the subspace spanned by the Laughlin wavefunctions.

Our simulations show a very good overlap with the Laughlin wavefunction for $\alpha \lesssim 0.3$, but the overlap start to fall off for $\alpha \gtrsim 0.3$. We emphasize that the excellent overlap with the Laughlin wavefunction is not accidental, e.g., for $N = 5$ and $\alpha \approx 0.24$ the size of the Hilbert space is $8.5 \cdot 10^5$ and the overlap is 95%. These numerical calculation therefore provide strong evidence that the Laughlin wavefunction captures the essential properties of the many-particle system. Experimentally it is desirable to have as large an excitation gap as possible. From the results in Fig. 2 (b) we see that the optimal regime is $\alpha \sim 0.2$ (although the data still have finite size effects). In this region the Laughlin wavefunction (1) is a very good description of the state.

In order to reach the Laughlin state experimentally it is necessary to adiabatically load a cold Bose-Einstein condensate (BEC) into the lattice. We expect the transition between the superfluid BEC and fractional quantum Hall states to be a direct first order phase transition or proceed via several intermediate phases [18]. Thus, a direct transition between the BEC and the fractional quantum Hall state is likely to create many excitations. To avoid this problem we suggest to accomplish the Laughlin state preparation through a Mott-insulator state [3, 4, 11]. Such a state with an integer number of atoms per site can be reached from a BEC by adiabatically raising the lattice potential so that U in Eq. (2) is much larger than J. We consider a situation where an additional (weak) super lattice $V = V_0(\sin^2(\pi x/p_x) + \sin^2(\pi y/p_y))$ is present, as realized experimentally in Ref. [19]. By loading a BEC into the combined potential it is then possible to reach a Mott insulator state with a single atom at each of the potential minima of the super lattice. In this state the atoms cannot move and are hence unaffected by the turn on of the effective magnetic field. By reducing the super

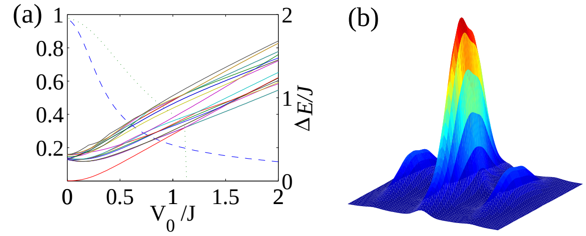

FIG. 3: (a) Creation of the Laughlin state by melting a Mott insulator. Full lines (right axis) energy relative to the ground state of the 19 lowest excited states for different amplitudes (V_0) of the super lattice potential. Dashed (dotted) line overlap of the ground state (first excited state) with the Laughlin wavefunction (1) (left axis). Note that at $V_0 = 0$ there are two nearly degenerate states because of the combination of magnetic field and periodic boundary conditions. These two states have a 98% overlap with the two possible Laughlin states. Results are for a 6×6 lattice with 4 atoms and 8 fluxes, $\alpha \approx 0.22$. The periods of the super lattice are $p_x = p_y = 3$. (b) Density distribution after expansion from the lattice (arbitrary units). For illustration we have assumed a lattice depth of 10 recoil energies [11] and $\alpha \approx 0.22$ as in (a).

lattice potential it is then possible to adiabatically reach the Laughlin state, as demonstrated in Fig. 3 (a). In the figure we show the evolution of the lowest energy levels when we reduce the strength of the super lattice potential V_0. For large values of V_0 the ground state is the Mott-insulator state which is well separated from all excited states. When $V_0 = 0$ there are two nearly degenerate ground states because of the center of mass degeneracy mentioned above. Apart from this mathematical artifact of the periodic boundary conditions, the ground state is always well separated from the excited states by an energy gap, and the ground state of the system smoothly changes into the Laughlin state, so that it is possible to reach the Laughlin state by adiabatically reducing the super lattice potential.

We next consider experimental issues involving the realization and detection of quantum Hall states in the lattice. Under realistic conditions the required "hard-core" limit can be reached, e.g., in atomic Rb with a tunneling rate $J/2\pi\hbar$ of the order of hundreds of Hz [3, 11], which indicates that energy gaps in the range 10-100 Hz can be obtained. To observe the quantum Hall states the temperature of the atoms need to below this excitation gap which requires realistic temperatures of a few nK. A limitation for the practical realization of the present proposal is that the oscillating potentials produce strong phase shifts on the atoms. If there is a total of N_ϕ fluxes in the lattices the phase shift on atoms in the outermost regions is on the order of N_ϕ per half-cycle. Hence a practical implementation requires strong gradients and it is necessary to ensure that the phase shift in one half of the pulse exactly balances the phase shift in the other.

Another limitation is that the oscillating quadrupole potential can excite higher Bloch bands. If we approximate the wells by a harmonic potential the weight on the excited state in the outermost regions of the lattice is $w_{\rm ex} \sim \pi N_\phi \alpha (a_0/\lambda)^2 \omega^2/\nu_b^2$, where ν_b is the Bloch band separation, and where the ground state width a_0 is of order $\lambda/10$ for typical lattice parameters [11]. With $\omega \sim \nu_b/10$ this is not a major concern for $N_\phi \lesssim 10^2 - 10^3$.

To demonstrate experimentally that one has reached the Laughlin states one would ideally probe some of the unique features of the state, such as incompressibility, the fractional charge of the excitations or their anyonic character. Such probes will most likely be very challenging to implement and we shall now discuss a simpler experimental indication. In most experiments with cold trapped atoms the state is probed by releasing the atoms from the trap and imaging the momentum distribution. To find the results of such an expansion we use the continuum wavefunction (1) which is a good description in the regime we are interested in. In the lowest Landau level the single particle density matrix for any state with constant density was found in Ref. [20]. From this density matrix we find the asymmetric expansion shown in Fig. 3 (b). This density distribution is clearly distinct from, e.g., a superfluid state which will have Bragg peaks, and a Mott-insulator, which gives a symmetric distribution [3]. This method, however, does not reveal detailed information about the state except that it is in the lowest Landau level. Further insight can be obtained by measuring higher order correlation functions [21, 22] or by measuring the excitation spectrum through stimulated Bragg scattering [23].

To summarize, we have presented a feasible method to construct fractional quantum Hall states in an optical lattice. Compared to previous proposals with cold atoms [6] the optical lattice approach results in a more robust quantum Hall state, since it is protected by a larger gap. The present approach therefore reduces the experimental requirement and could facilitate the observation of such states with a larger number of particles.

Several interesting new avenues are opened by this work. First of all, it would be interesting to understand the exact nature of the ground state when there is a large flux fraction per unit cell $\alpha \gtrsim 0.3$. We have not made any modification to the Laughlin wavefunction to take the lattice into account, and one would expect this to reduce the overlap in Fig. 2 (a) when α is not vanishingly small. The decrease in the overlap is, however, much less abrupt for the single particle wavefunctions. We therefore do not expect that this can explain the observed results. Alternatively the decrease could be caused by the system entering a different phase for $\alpha \gtrsim 0.3$, e.g., a superfluid state with a vortex lattice. This possibility is supported by the decrease in the excitation gap above $\alpha \gtrsim 0.25$, see Fig. 2 (b). We further observe an increase in the largest eigenvalue of the one-particle density matrix above $\alpha \gtrsim 0.3$ (especially for $\alpha = 1/3$ and $\alpha = 1/2$ that correspond to a commensurate density of vortices) which is consistent with a superfluid state. We cannot, however, draw any definite conclusion from our present numerical results. In addition the present method for creating the quantum Hall state in the lattice can be easily extended to yield different magnetic field for different internal atomic states of multi-component bosons. Using this approach effective non-abelian gauge fields can be created. Therefore the present method may allow one to explore experimentally the novel properties of a many particle system in the presence of such a field.

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